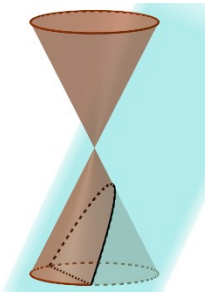
Precalculus

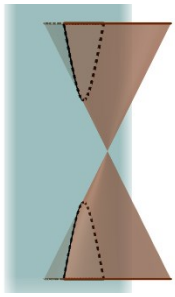
7-01 Lines

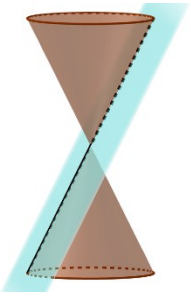
Conic sections

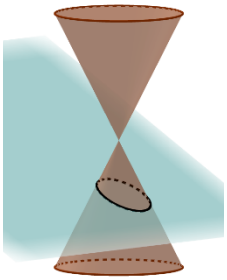
- Intersections of a _____with a _____
- _____
- _____
- _____

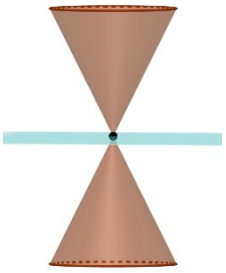


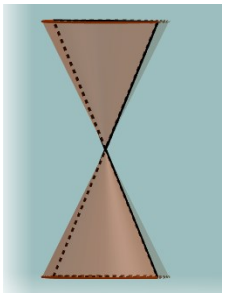
- _____
- _____
- _____





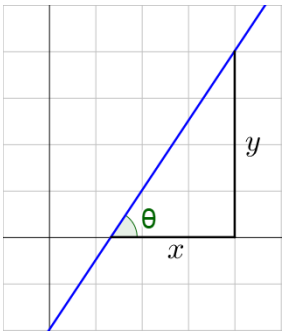






Lines

- $Ax + By + C = 0$ or $y = mx + b$



Inclination

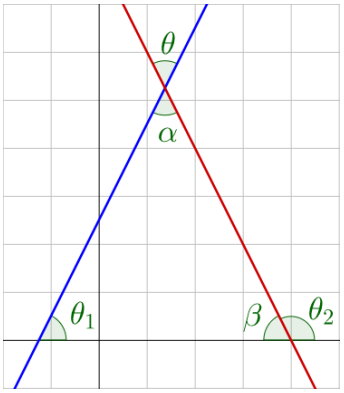
- Describes _____of line
- Angle it makes with _____

$$\tan\theta = m$$

- Where $0° < \theta < 180°$
  - If $\theta < 0$, add 180°

Find the inclination of $4x - 2y + 5 = 0$.

Angle between Two Lines



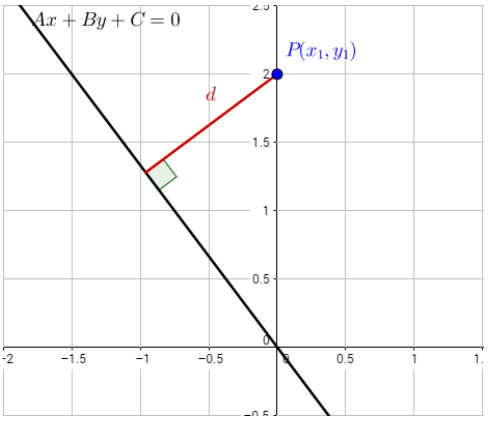
$$\theta = \theta_2 - \theta_1$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Where $0° < \theta < 90°$

Find the angle between $2x + y = 4$ and $x - y = 2$.

Distance from a Point to a Line



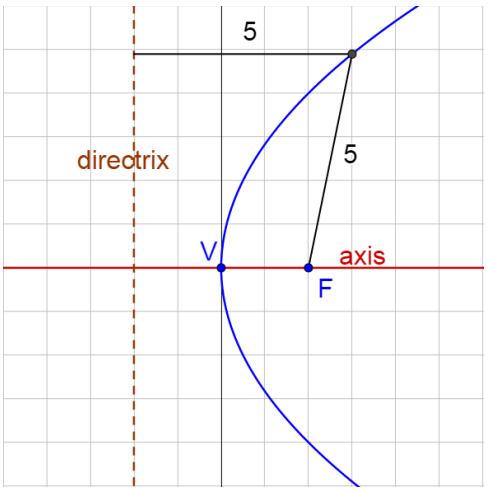
- Point $(x_1,\ y_1)$ and Line $Ax + By + C = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Find the distance from $(0, 2)$ to $4x + 3y = 0$.

Precalculus

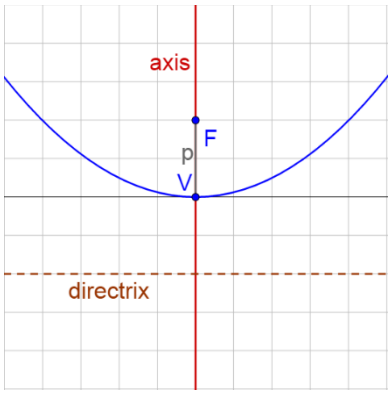
7-02 Parabolas



Parabolas

- Set of all points in a plane that are _____from a fixed line, called the _____and a fixed point, called the _____.
- Vertex
  - max or min point
  - midpoint between the _____and _____.
- Axis of symmetry
  - line perpendicular to the _____
  - goes through the _____and _____.
- Parabola bends _____the focus and _____from the directrix.

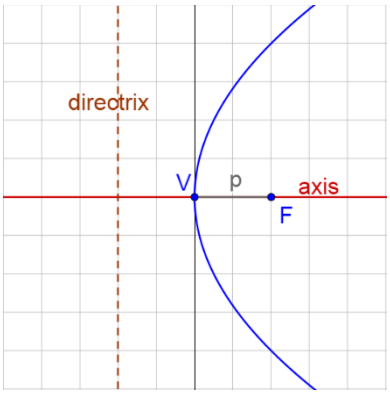
Vertical Parabola



- p = directed (+, −) distance from vertex to focus
- Vertex (h, k)
- Focus $(h,\ p + k)$
- Directrix $y = k - p$

$$(x - h)^2 = 4p(y - k)$$

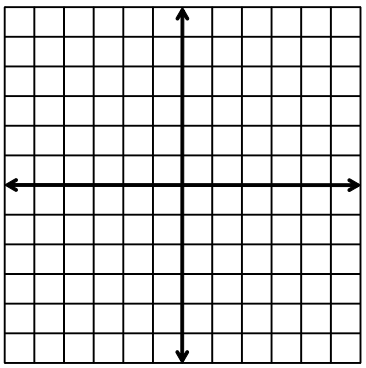
Horizontal Parabola



- p = directed (+, −) distance from vertex to focus
- Vertex (h, k)
- Focus $(p + h,\ k)$
- Directrix $x = h - p$

$$(y - k)^2 = 4p(x - h)$$

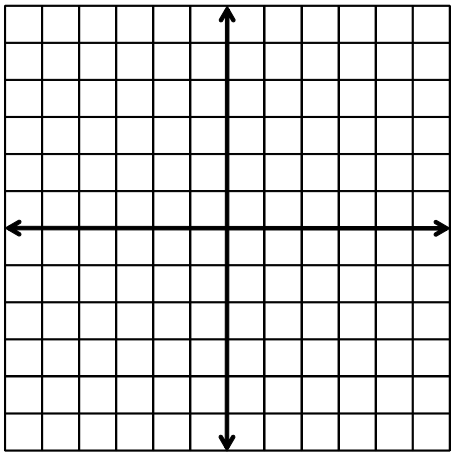
Find the vertex, focus, and directrix of the parabola given by $y = \frac{1}{2}x^2$.



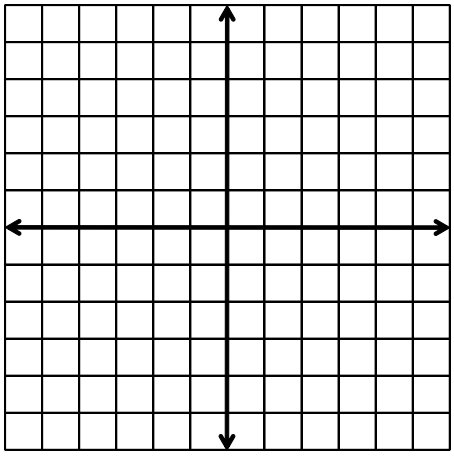
Find the standard form of the equations of a parabola with vertex at (0, 0) and focus (−2, 0).

Find the vertex, focus, and directrix of the parabola given by $x^2 - 2x - 16y - 31 = 0$.

Graph $(x-1)^2 = 16(y+2)$



Write the standard form of the equation of the parabola with focus (1, 2) and directrix $x = 3$.



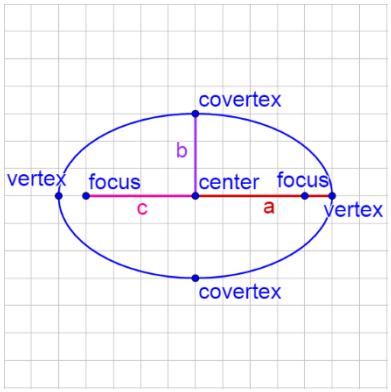
Precalculus

7-03 Ellipses and Circles

Ellipse

- Set of all points in a plane where the sum of the _____to two fixed points, _____, is constant.
- Major axis
  - _____segment across the ellipse
  - Connects the two _____.
- Minor axis
  - _____segment across the ellipse
  - Connects the two _____.
- Circle
  - Special form of an ellipse where both foci are at the _____.

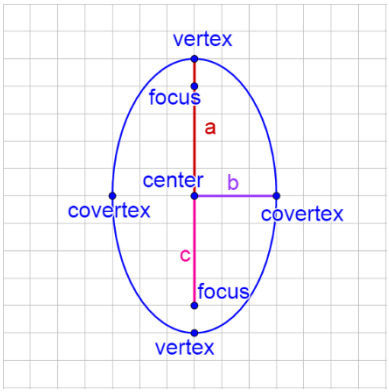
Horizontal Ellipse



- Center $(h,\ k)$
- Horizontal Major Axis length = $2a$
- Vertical Minor Axis length = $2b$
- $c^2 = a^2 - b^2$
- Vertices $(h \pm a,\ k)$
- Covertices $(h,\ k \pm b)$
- Foci $(h \pm c,\ k)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertical Ellipse



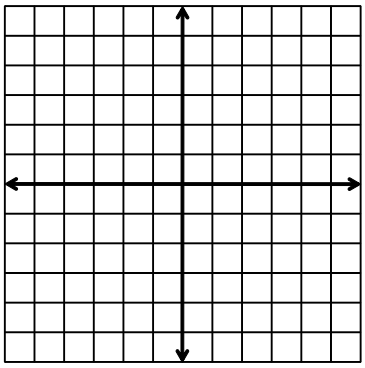
- Center $(h,\ k)$
- Vertical Major Axis length = $2a$
- Horizontal Minor Axis length = $2b$
- $c^2 = a^2 - b^2$
- Vertices $(h,\ k \pm a)$
- Covertices $(h \pm b, k)$
- Foci $(h,\ k \pm c)$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

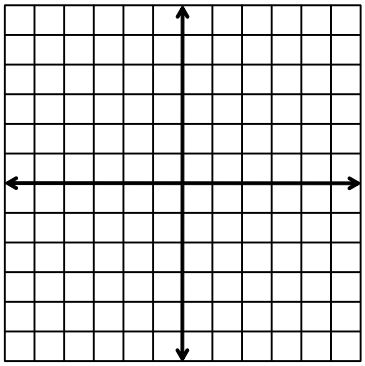
a = distance from center to _____
b = distance from center to _____
c = distance from center to _____

Find the center, vertices, and foci of the ellipse $9x^2 + 4y^2 = 36$.

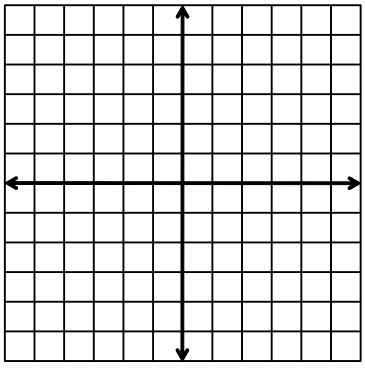
Find the standard form of the ellipse centered at (1, 2) with major axis length 10 and foci (−2, 2) and (4, 2).



Graph $\frac{(x-1)^2}{25}+\frac{(y-2)^2}{16}=1$



Sketch the graph of $25x^2+9y^2-200x+36y+211=0$



Eccentricity

- Measure of how _____an ellipse is
- $e=\frac{c}{a}$ where $0<e<1$
- If $e\approx 0$, then ellipse is almost a _____
- If $e\approx 1$, then ellipse is almost a _____

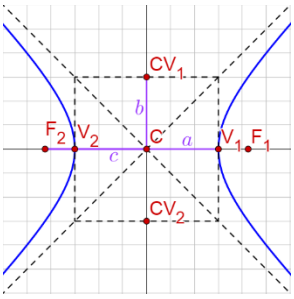
Precalculus

7-04 Hyperbolas

Set of all points in a plane where the _____of the distances from two set points, _____, is constant.

- $d_1 - d_2$ = _____.

Horizontal Hyperbola

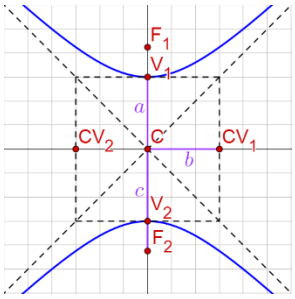


- Center $(h,\ k)$
- Horizontal Transverse Axis length = $2a$
- Vertical Conjugate Axis length = $2b$
- $c^2 = a^2 + b^2$
- Vertices $(h \pm a,\ k)$, Covertices $(h,\ k \pm b)$
- Foci $(h \pm c,\ k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

- Asymptotes $y = k \pm \frac{b}{a}(x-h)$

Vertical Hyperbola



- Center $(h,\ k)$
- Vertical Transvers Axis length = $2a$
- Horizontal Conjugate Axis length = $2b$
- $c^2 = a^2 + b^2$
- Vertices $(h,\ k \pm a)$, Covertices $(h \pm b, k)$
- Foci $(h,\ k \pm c)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

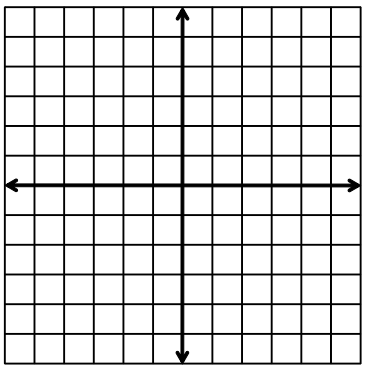
- Asymptotes $y = k \pm \frac{a}{b}(x-h)$

Eccentricity

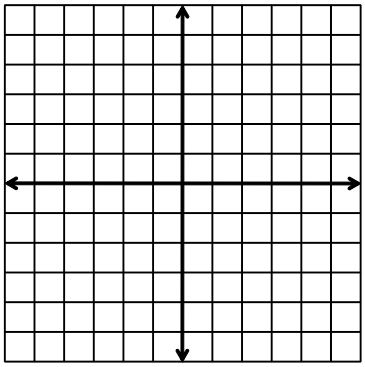
- $e = \frac{c}{a}$, where $e > 1$
- Big e = _____branches

Find the center, vertices, asymptotes, and foci of the hyperbola $4y^2 - 9x^2 = 36$.

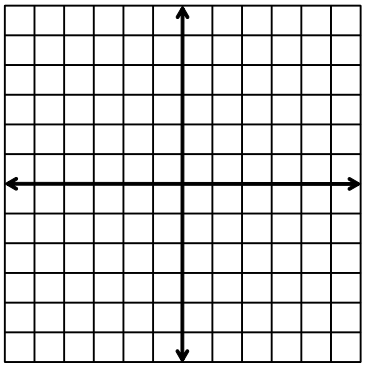
Find the standard form of the hyperbola centered at (1, 2) with transverse axis length 10 and foci (−5, 2) and (7, 2).



Graph $\frac{(x-1)^2}{25} - \frac{(y-2)^2}{11} = 1$



Sketch the graph of $4x^2 - 9y^2 - 24x - 72y - 72 = 0$



General form of conics

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

- Circle if $A = C$
- Parabola if $AC = 0$ (so $A = 0$ or $C = 0$)
- Ellipse if $AC > 0$
- Hyperbola if $AC < 0$

Classify the conics

$4x^2 + 5y^2 - 9x + 8y = 0$

$2x^2 - 5x + 7y - 8 = 0$

$7x^2 + 7y^2 - 9x + 8y - 16 = 0$

$4x^2 - 5y^2 - x + 8y + 1 = 0$

Precalculus

7-05 Rotated Conics

- Nonrotated conics form $Ax^2 + Cy^2 + Dx + Ey + F = 0$.
  - _____horizontal or vertical.
- Rotated conics form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.
  - _____horizontal or vertical
  - Bxy term prevents _____the _____to write the conics in standard form.

Classify Rotated Conics

- If the conic is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, then
  - If $B^2 - 4AC < 0 \rightarrow$ _____
  - If $B^2 - 4AC = 0 \rightarrow$ _____
  - If $B^2 - 4AC > 0 \rightarrow$ _____

Write Rotated Conics in Standard Form

Given a conic written as $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

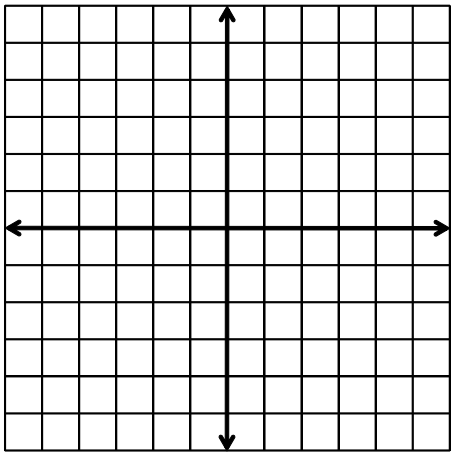
1. Find the angle of rotation using
$$\mathbf{\cot 2\theta = \frac{A - C}{B}}$$
where $0 < \theta < \frac{\pi}{2}$
2. Find $\sin\theta$ and $\cos\theta$.
   - If θ is a special angle, evaluate $\sin\theta$ and $\cos\theta$ directly.
   - If θ is not a special angle,
   a. Find $\cot 2\theta$.
   b. Reciprocal to find $\mathbf{\tan 2\theta}$.
   c. Use $\mathbf{1 + \tan^2 u = \sec^2 u}$ to find $\sec 2\theta$. (If $\tan 2\theta < 0$, then $\sec 2\theta < 0$.)
   d. Reciprocal to find $\mathbf{\cos 2\theta}$.
   e. Use the half-angle formulas to find $\sin\theta$ and $\cos\theta$.
   $$\mathbf{\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}} \text{ and } \mathbf{\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}}$$
3. Find the substitutions for x and y using
$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$
4. Make the substitutions and arrange the terms into standard form.

Graph a Rotated Conic

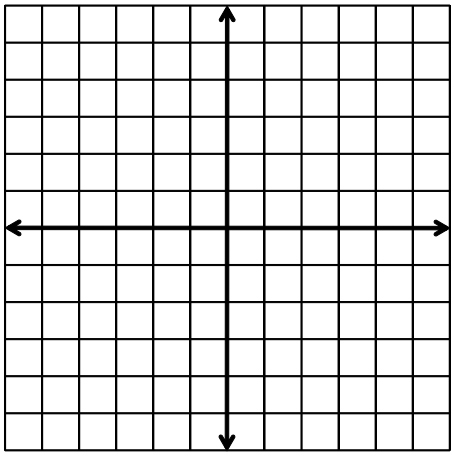
1. Draw the rotated _____.
2. Using the rotated axes, sketch the _____.

Write $xy = \frac{1}{2}$ in standard form

Sketch the graph of $x^2 + \sqrt{3}xy + 2y^2 - 2 = 0$.

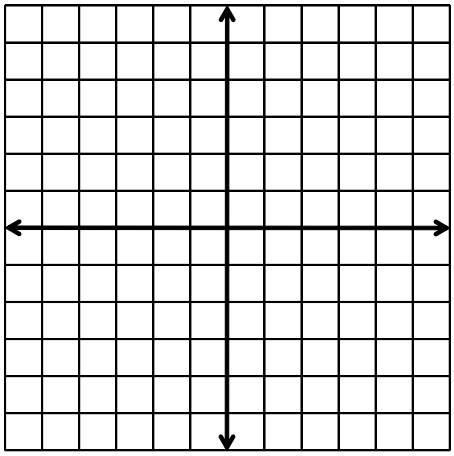


Sketch the graph of $3x^2 + 2\sqrt{3}xy + y^2 + 2x - 2\sqrt{3}y = 0$.



Classify the graph, use the quadratic formula to solve for y, and use a graphing utility to graph the equation.

$3x^2 - 6xy + 3y^2 + 2y = 0$



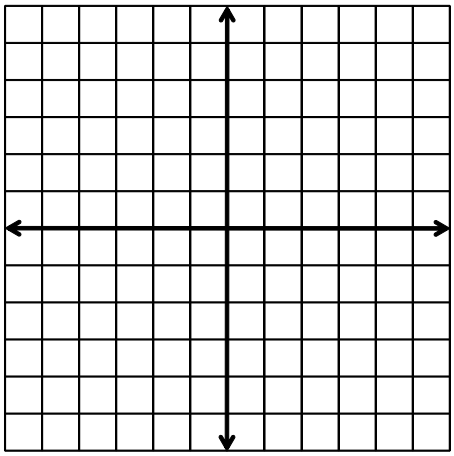
Precalculus

7-06 Parametric Equations

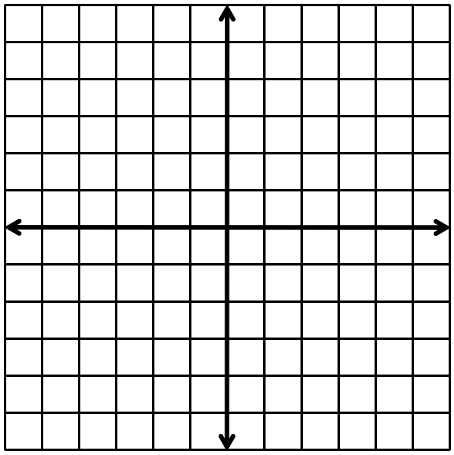
Parametric Equations

- Separate _____for x and y
- x and y are functions of a _____variable called a _____

Graph $\begin{cases} x = t - 3 \\ y = t^2 + 1 \end{cases}$



Graph $\begin{cases} x = 2\cos\theta \\ y = 2\sin\theta \end{cases}$ for $0 \le \theta \le 2\pi$



Eliminating the Parameter

1. Solve one equation for _____
2. _____it into the other equation

Eliminate the parameter of $\begin{cases} x = \frac{1}{\sqrt{t}} \\ y = 2t^2 \end{cases}$

Eliminate the parameter in $\begin{cases} x = 2\cos\theta \\ y = 2\sin\theta \end{cases}$

Finding parametric equations

1. Choose something _____to equal _____

Find parametric equations for $y = 4x - 3$

Find parametric equations for conics.

Parabola

- Horizontal: $\begin{cases} x = pt^2 + h \\ y = 2pt + k \end{cases}$
- Vertical: $\begin{cases} x = 2pt + h \\ y = pt^2 + k \end{cases}$

Ellipse

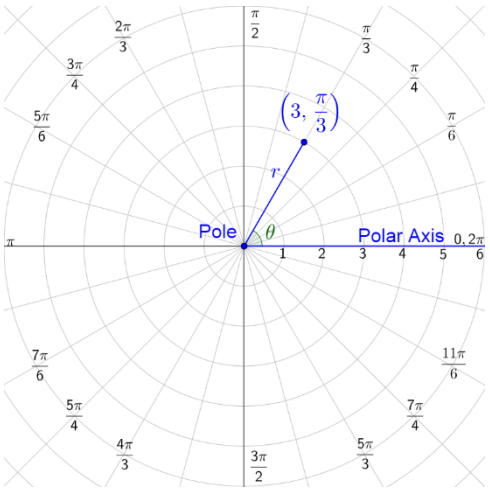
- Horizontal: $\begin{cases} x = h + a\cos t \\ y = k + b\sin t \end{cases}$
- Vertical: $\begin{cases} x = h + b\sin t \\ y = k + a\cos t \end{cases}$

Hyperbola

- Horizontal: $\begin{cases} x = h + a\sec t \\ y = k + b\tan t \end{cases}$
- Vertical: $\begin{cases} x = h + b\tan t \\ y = k + a\sec t \end{cases}$

Precalculus

7-07 Polar Coordinates



Polar coordinates

- (r, θ)
  - r = _____from _____
  - θ = angle _____from _____axis

Graph

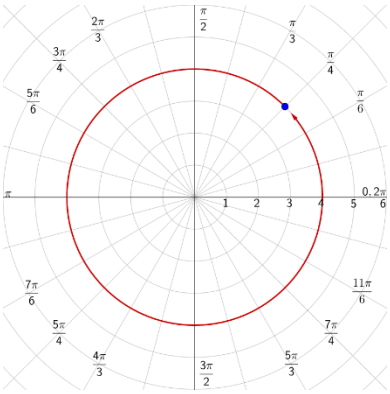
$A\left(4, \frac{\pi}{4}\right)$

$B\left(-5, \frac{2\pi}{3}\right)$

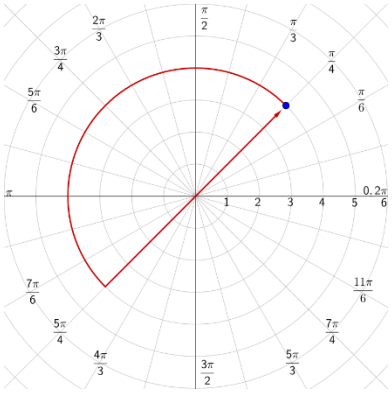
$C\left(3, -\frac{\pi}{6}\right)$

Multiple ways to represent same point

- Add _____circles
  - $(r, \theta) = (r, \theta \pm 2\pi n)$



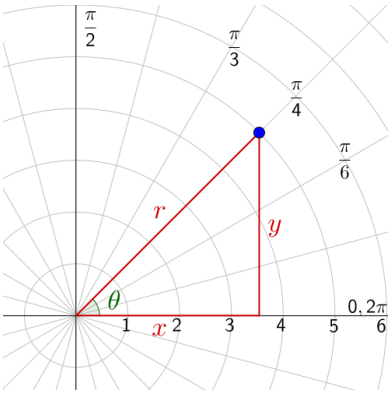
- _____side of circle and add _____ circle
  - $(r, \theta) = (-r, \theta \pm (2n+1)\pi)$



Find 2 other ways to write $\left(3, \frac{\pi}{4}\right)$.

Precalculus 7-07 Name: _____

Convert between polar and rectangular



Polar → Rectangular

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Rectangular → Polar

$$r = \sqrt{x^2 + y^2}$$
$$\tan\theta = \frac{y}{x}$$

Convert $\left(4, \frac{\pi}{6}\right)$ to rectangular

Convert $(-1,\ 0)$ to polar

Convert Equations

Convert $r = 1$

Convert $\theta = \frac{\pi}{4}$

Convert $r = \csc\theta$

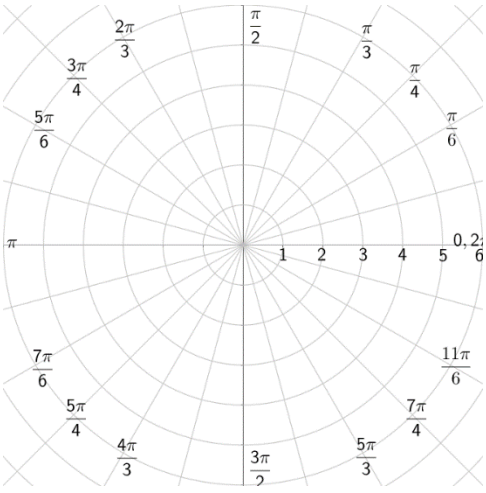
Precalculus

7-08 Graphs of Polar Equations

To Graph Polar Equations Using a Table

- Pick _____and calculate _____

Graph $r = 3\cos\theta$



Symmetry Tests (make the replacement and to simplify to original equation)

- Line $\theta = \frac{\pi}{2}$
  - Replace (r, θ) with $(r, \pi - \theta)$ or $(-r, -\theta)$
- Polar Axis
  - Replace (r, θ) with $(r, -\theta)$ or $(-r, \pi - \theta)$
- Pole
  - Replace (r, θ) with $(r, \pi + \theta)$ or $(-r, \theta)$
- Quick tests
  - If it is a function of _____, then _____ symmetry
  - If it is a function of _____, then _____ symmetry

Find the symmetry of $\theta = \frac{\pi}{4}$

Find the symmetry of $r = 2(1 - \sin\theta)$

Maximums and Zeros of Polar Equations

- Maximums occurs when _____ is largest.
    - Find angles where the trigonometric function is at its _____.
- Zeros occur when _____.
    - Find angles where the trigonometric function is _____.

Find the zeros and maximum r values of $r = 5\cos 2\theta$

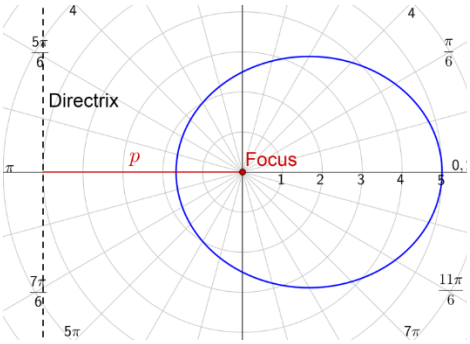
Precalculus

7-09 Polar Graphs of Conics

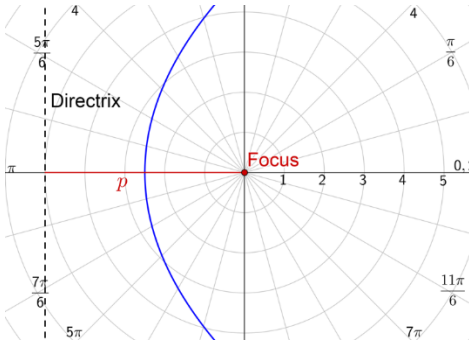
Alternative Definition of a Conic Section

- Locus of a point in the plane that moves so its distance from a fixed _____ (_____) is in a constant ratio to its distance from a fixed _____(_____).
- The ratio is the _____(*e*).

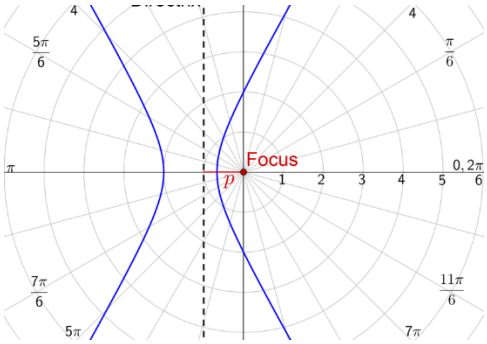
$e < 1$ ellipse



$e = 1$ parabola



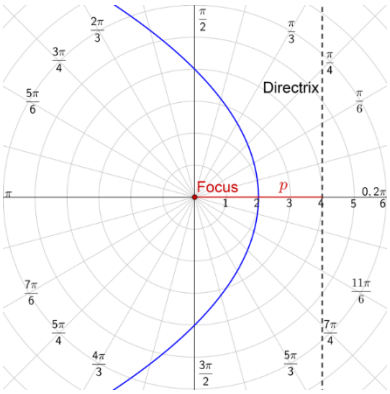
$e > 1$ hyperbola



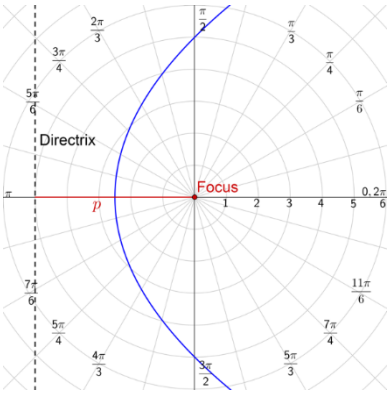
- *p* = distance from _____to _____
- One focus is _____
- The conic bends _____the focus and _____from directrix

Vertical Directrix

Right of pole: $r = \frac{ep}{1+e\cos\theta}$

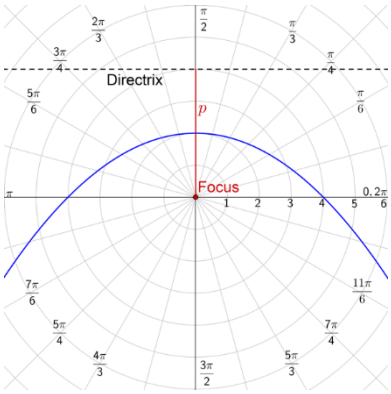


Left of pole: $r = \frac{ep}{1-e\cos\theta}$

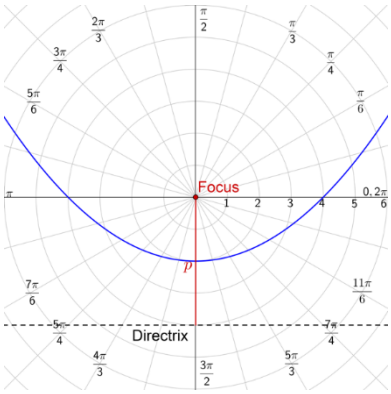


Horizontal Directrix

Above pole: $r = \frac{ep}{1+e\sin\theta}$

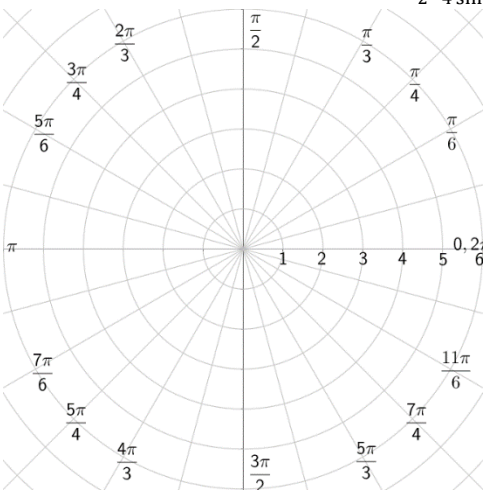


Below pole: $r = \frac{ep}{1-e\sin\theta}$

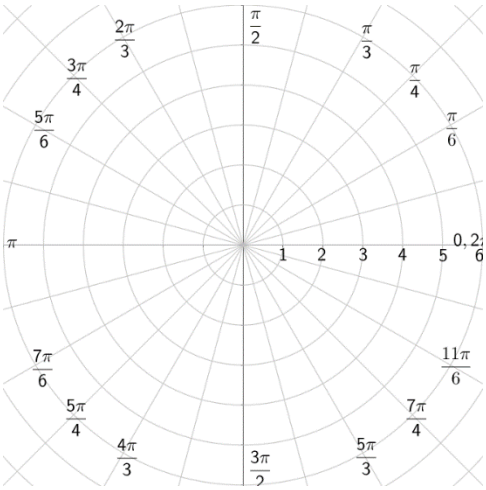


Identify the type of conic $r = \frac{2}{2+\cos\theta}$

Identify type of conic and graph $r = \frac{3}{2-4\sin\theta}$



Find the polar equation of the parabola whose focus is the pole and directrix is the line $x = -2$.



Find the polar equation of the hyperbola with focus at pole and vertices $\left(1, \frac{3\pi}{2}\right)$ and $\left(-9, \frac{\pi}{2}\right)$.