

Mr. Wright's Math Extravaganza

Precalculus Conic Sections

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications Score I Can Statements

Score	
4.0	I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content
	I can write and graph conic functions in rectangular coordinates.
2.0	I can classify conics by their equations.
5.0	I can write and graph parametric functions in parametric form.
	 I can write and graph conic functions in polar form.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content
	I can find the inclination of a line.
	 I can find the angle between two lines.
	I can write the equation, graph, and find key parts of a parabola.
	I can write the equation, graph, and find key parts of an ellipse.
2.0	I can write the equation, graph, and find key parts of a hyperbola.
	I can write the rotated conics in rectangular coordinates.
	 I can graph and write parametric equations.
	I can convert between polar and rectangular coordinates.
	 I can graph polar coordinates and equations.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

7-01 Lines

Conic sections		
Intersections of a	with a	
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Lines		
• $Ax + By + C = 0$ or $y = m$	x + b	
Inclination		
Describes	of line	
Angle it makes with		
	$\tan \theta = m$	
• where $0^\circ < \theta < 180^\circ$		
Find the inclination of $4x - 2y + 5 =$	0.	





Find the vertex, focus, and directrix of the parabola given by $x^2 - 2x - 16y - 31 = 0$.

Graph $(x - 1)^2 = 16(y + 2)$

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Writ	e th	ie st	tan	dar	d fo	orm	of	the	eqı	ıati	on	of the parabola with focus (1, 2) and directrix $x = 3$.
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7-03 Ellipses and Circles



Find the standard form of the ellipse centered at (1, 2) with major axis length 10 and foci (-2, 2) and (4, 2).

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Eccentricity

- Measure of how _____an ellipse is
- $e = \frac{c}{a}$ where 0 < e < 1
- If $e \approx 0$, then ellipse is almost a _____
- If $e \approx 1$, then ellipse is almost a _____

7-04 Hyperbolas



Find the standard form of the hyperbola centered at (1, 2) with transverse axis length 10 and foci (-5, 2) and (7, 2).



Name: _____



Sketch the graph of
$$4x^2 - 9y^2 - 24x - 72y - 72 = 0$$



General form of conics

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

- Circle if A = C
- Parabola if AC = 0 (so A = 0 or C = 0)
- Ellipse if AC > 0
- Hyperbola if *AC* < 0

Classify the conics

 $4x^2 + 5y^2 - 9x + 8y = 0$

 $2x^2 - 5x + 7y - 8 = 0$

 $7x^2 + 7y^2 - 9x + 8y - 16 = 0$

$$4x^2 - 5y^2 - x + 8y + 1 = 0$$

7-05 Rotated Conics

- Nonrotated conics form $Ax^2 + Cy^2 + Dx + Ey + F = 0$.
 - _____horizontal or vertical.
- Rotated conics form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.
 - _____horizontal or vertical
 - *Bxy* term prevents ______the _____to write the conics in standard form.

Classify Rotated Conics

- If the conic is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, then
 - $\circ \quad \text{If } B^2 4AC < 0 \rightarrow ____$
 - $\circ \quad \text{If } B^2 4AC = 0 \rightarrow ____$
 - $\circ \quad \text{If } B^2 4AC > 0 \rightarrow ___$

Write Rotated Conics in Standard Form

Given a conic written as $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

1. Find the angle of rotation using

$$\cot 2\theta = \frac{A-C}{B}$$

where $0 < \theta < \frac{\pi}{2}$

- 2. Find sin θ and cos θ .
 - If θ is a special angle, evaluate sin θ and cos θ directly.
 - If θ is not a special angle,
 - a. Find $\cot 2\theta$.
 - b. Reciprocal to find **tan 2***θ*.
 - c. Use $1 + \tan^2 u = \sec^2 u$ to find sec 2θ . (If $\tan 2\theta < 0$, then sec $2\theta < 0$.)
 - d. Reciprocal to find **cos** 2*θ*.
 - e. Use the half-angle formulas to find $\sin \theta$ and $\cos \theta$.

$$\sin \theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$
 and $\cos \theta = \sqrt{\frac{1+\cos 2\theta}{2}}$

3. Find the substitutions for *x* and *y* using

$$x = x' \cos \theta - y' \sin \theta$$
$$y = x' \sin \theta + y' \cos \theta$$

4. Make the substitutions and arrange the terms into standard form.

Graph a Rotated Conic

- 1. Draw the rotated _____
- 2. Using the rotated axes, sketch the _____.

Write $xy = \frac{1}{2}$ in standard form

Sketch the graph of $x^2 + \sqrt{3xy} + 2y^2 - 2 = 0$

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Name: ____

Precalculus 7-05

Sketch the graph of $3x^{2} + 2\sqrt{3}xy + y^{2} + 2x - 2\sqrt{3}y = 0$.

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Precalculus 7-05

Classify the graph, use the quadratic formula to solve for y, and use a graphing utility to graph the equation. $3x^2 - 6xy + 3y^2 + 2y = 0$

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7-06 Parametric Equations



Eliminating the Parameter

- 1. Solve one equation for _____
- 2. _____it into the other equation

Eliminate the parameter of
$$\begin{cases} x = \frac{1}{\sqrt{t}} \\ y = 2t^2 \end{cases}$$

Eliminate the parameter in
$$\begin{cases} x = 2\cos\theta \\ y = 2\sin\theta \end{cases}$$

Finding parametric equations

1. Choose something _

_to equal

Find parametric equations for y = 4x - 3

Find parametric equations for conics.

Parabola

- Horizontal: $\begin{cases} x = pt^{2} + h \\ y = 2pt + k \end{cases}$ Vertical: $\begin{cases} x = 2pt + h \\ y = pt^{2} + k \end{cases}$ ٠

Ellipse

- ٠
- Horizontal: $\begin{cases} x = h + a \cos t \\ y = k + b \sin t \end{cases}$ Vertical: $\begin{cases} x = h + b \sin t \\ y = k + a \cos t \end{cases}$ •

Hyperbola

- Horizontal: $\begin{cases} x = h + a \sec t \\ y = k + b \tan t \end{cases}$ Vertical: $\begin{cases} x = h + b \tan t \\ y = k + a \sec t \end{cases}$





7-08 Graphs of Polar Equations

$araph r = 3 \cos \theta$ $\frac{2\pi}{3\pi}$ $\frac{\pi}{2}$				
2π $\frac{\pi}{2}$ $\frac{\pi}{2}$ 3π 3π				
$ \begin{array}{c} 4 \\ 5\pi \\ 6 \\ 7\pi \\ 6 \\ 5\pi \\ 4 \\ 4\pi \\ 3 \\ 3\pi \\ 2 \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			
ymmetry Tests (make	the replacement an	d to simplify to origin	al equation)	
• Line $\theta = \frac{\pi}{2}$				
• Replace	$e(r, \theta)$ with $(r, \pi - \theta)$	or $(-r, -\theta)$		
Polar Axis				
 Replace 	$e(r, \theta)$ with $(r, -\theta)$	or $(-r, \pi - \theta)$		
• Pole				
 Replace 	$e(r, \theta)$ with $(r, \pi + \theta)$) or (<i>−r, θ</i>)		
Quick tests				
∘ If it is a	function of	, then	symmetry	
 If it is a 	function of	, then	symmetry	
ind the symmetry of θ	$=\frac{\pi}{2}$			
	4			

Find the symmetry of $r = 2(1 - \sin \theta)$

Precalculus 7-08	Name:
Maximums and Zeros of Polar Equations	

_____.

- Maximums occurs when ______ is largest.
 - Find angles where the trigonometric function is at its _____.
- Zeros occur when ______
 - Find angles where the trigonometric function is ____

Find the zeros and maximum *r* values of $r = 5 \cos 2\theta$

7-09 Polar Graphs of Conics



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Identify the type of conic $r = \frac{2}{2 + \cos \theta}$



Find the polar equation of the parabola whose focus is the pole and directrix is the line x = -2.



Find the polar equation of the hyperbola with focus at pole and vertices $\left(1,\frac{3\pi}{2}\right)$ and $\left(-9,\frac{\pi}{2}\right)$.

